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Towards the string representation of the dual abelian Higgs model beyond the London limit

Yoshiaki Koma
Institute for Theoretical Physics, Kanazawa University
Kanazawa 920-1192, Japan
E-mail: koma@hep.s.kanazawa-u.ac.jp

Miho Koma (Takayama), Dietmar Ebert* and Hiroshi Toki
Research Center for Nuclear Physics, Osaka University
Mihogaoka 10-1, Ibaraki, Osaka 567-0047, Japan
E-mail: takayama@rcnp.osaka-u.ac.jp, ebert@rcnp.osaka-u.ac.jp, toki@rcnp.osaka-u.ac.jp

Abstract: We perform a path-integral analysis of the string representation of the dual abelian Higgs (DAH) model beyond the London limit, where the string describing the vortex of a flux tube has a finite thickness. We show that besides an additional vortex core contribution to the string tension, a modified Yukawa interaction appears as a boundary contribution in the type-II dual superconducting vacuum. In the London limit, the modified Yukawa interaction is reduced to the Yukawa one.

Keywords: Confinement, Duality in Gauge Field Theories, QCD.

*On leave of absence from Institute of Physics, Humboldt University, Berlin

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Contents

1. Introduction

The construction of a realistic low-energy lagrangian for hadrons based on quantum chromodynamics (QCD) clearly requires a deeper understanding of the mechanism of confinement. A very useful concept for an analytical description of this phenomenon is the dynamical scheme of a dual superconductor proposed more than twenty years ago by 't Hooft and Mandelstam [1, 2]. This approach emphasizes, in particular, the role of magnetic monopoles for confinement. The condensation of monopoles squeezes the chromoelectric flux into (open) Abrikosov-Nielsen-Olesen (ANO) type vortices [3, 4, 5], which then confine the quark and antiquark sitting at their ends. Recent studies in lattice QCD in the maximally abelian gauge indeed suggest remarkable properties of the QCD vacuum, such as abelian dominance [6, 7] and monopole condensation [8], which confirm the dual superconductor picture, suitably described by a dual abelian Higgs (DAH) model representing just a corresponding dual Ginzburg-Landau type of theory [9, 10]. The DAH model is here obtained by abelian projection [11] which is a crucial step in order to find the relevant IR degrees of freedom of QCD.

In the present paper we are mainly interested in analytical studies of the confinement mechanism leading to a path-integral derivation of effective string actions from the SU(2_c)-DAH model. The corresponding path-integral approach is essentially simplified in the so-called London limit of large monopole self-coupling $\lambda \to \infty$ (large monopole mass $m_\chi \to \infty$), where the modulus of the magnetic monopole field is frozen to its v.e.v., and ANO vortices become infinitely thin (core radius $\rho = (m_\chi)^{-1} \to 0$). In this limit the DAH model can be suitably reformulated as a theory of a massive antisymmetric Kalb-Ramond (KR) field interacting with surface elements of the world-sheet swept out by the ANO vortex (Dirac string). By performing finally a derivative expansion of the resulting effective action, it was possible to derive the Nambu-Goto action [12]–[15] including a correction (rigidity) term [16, 17], to estimate field strength correlators of the DAH model [18, 19] and comparing them with corresponding quantities of the Stochastic Vacuum Model of QCD [20, 21, 22]. Obviously, the London limit picture is only appropriate for large transverse distances from the vortex (string), where the thickness of the core and the corresponding contribution to the field energy per unit length (string tension) are neglected [12]–[15], [18]. Moreover, since the monopole field in this approximation is nowhere vanishing, one gets a massive dual gluon leading to a Yukawa interaction term in addition to the confining potential. Clearly, it is a challenge to go beyond the London limit in the sense of taking into account the finite thickness of vortices and the vanishing of the monopole field inside the vortex core. One might then naturally ask, whether one gets besides of the confining potential a Coulomb potential (as used in quarkonium spectroscopy) instead of the Yukawa one, or possibly something between them [19].

The main goal of this paper is an attempt to extend the usual path-integral approach as much as possible beyond the London limit, paying special attention to the treatment of boundary terms related to the nonconfining (shorter range) part of the potential. For this aim and also by pedagogical reasons, we find it convenient to use throughout differential form techniques allowing for a transparent and compact treatment (for definitions, see table 1 and 2). As in earlier works (see e.g. [9, 10, 15, 18]), the interaction with an external $q\bar{q}$-pair is introduced into the dual field strength F of the DAH model in the form of an (open) external electric Dirac string $\Sigma^{\rm open}$ as, $F = dB - 2\pi * \Sigma^{\rm open}$, so that the dual Bianchi identity is broken, $dF \neq 0$. Differing from the above quoted papers, we find it, however, convenient to decompose the dual gauge potential B into a regular and a singular part [23, 24, 25] so that the singular part cancels the Dirac string in dF leaving a Coulomb term which keeps the broken dual Bianchi identity intact. Moreover, for performing the path-integration over the KR field, special emphasize has to be paid to the corresponding gauge condition. As our analysis shows, for a rough estimate based on a space time "mesh size" larger than the coherence length m_χ^{-1} (chosen as the inverse of the effective cut-off in the momentum integrals), the London limit estimate of the effective string action resulting from field distributions outside the string core remains approximately valid. Clearly, in this case, we cannot see the inside of the ANO vortex, whose contribution has to be estimated separately by using classical field equations of motions. Below we only quote this expression without doing a numerical estimate (this would first require to fix the DAH parameters from lattice data). The main result of the paper is an effective string action from which one gets a string tension given as a sum of a core and a gauge field ("vortex surface") contribution, and a nonconfining potential having the form of a modified Yukawa interaction. Obviously, a more complete study of the DAH model, as considered here, would require the inclusion of quantum fluctuations of the string [26, 27, 28] which is, however, outside the scope of this analysis.

The organization of the paper is the following. In section 2 we formulate the DAH model in differential forms using a field decomposition with cancellation of the Dirac string in the dual field strength. In section 3 the path-integral approach is discussed paying special attention to the gauge fixing condition for the KR field and boundary terms related to nonconfining electric current interactions. Section 4 is devoted to the derivation of the effective action, and section 5 contains conclusions.

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r-form ($0 \leq r \leq 4$)	ω	$\omega \equiv \frac{1}{r!}\omega_{\mu_1 \ldots \mu_r} dx_{\mu_1} \wedge \cdots \wedge dx_{\mu_r}$
exterior derivative	d	r-form $\mapsto$ $(r+1)$-form
Hodge star	$*$	r-form $\mapsto$ $(4-r)$-form
	$**$	multiply a factor $(-1)^r$ for r-form
codifferential	$\delta \equiv - * d *$	r-form $\mapsto$ $(r-1)$-form
Laplacian	$\Delta \equiv d\delta + \delta d$	r-form $\mapsto$ r-form
Inner product	$(\omega, \eta) \equiv \int \omega \wedge * \eta$	$(\omega, \eta) = \frac{1}{r!}\int d^4x \omega_{\mu_1 \ldots \mu_r} \eta_{\mu_1 \ldots \mu_r}$ $(\omega, \eta \in r\text{-form})$ $(\omega)^2 \equiv (\omega, \omega)$

Table 1: Definitions in differential forms in the four-dimensional euclidean space-time.

2. The dual abelian Higgs model

In this section, we formulate the DAH model using differential form techniques (see, tables 1 and 2) [29]. Let the dual gauge field and the complex scalar monopole field be B (1-form) and $\chi = \phi \exp(i\eta)$ (0-form), respectively, then, the DAH model with an external electric Dirac string Σ^{open}, whose ends are electric charges q and $\bar{q}$, in euclidean space-time is given by

$$S\left(B, \chi, \Sigma^{\text{open}}\right) = \frac{\beta_g}{2}(F)^2 + (d\phi)^2 + ((B + d\eta)\phi)^2 + \lambda\left(\phi^2 - v^2\right)^2 , \tag{2.1}$$

where the dual field strength F is expressed as

$$F = dB - 2\pi * \Sigma^{\text{open}} . \tag{2.2}$$

Due to the presence of the electric Dirac string Σ^{open} (2-form), the dual Bianchi identity is broken as

$$dF = -2\pi d * \Sigma^{\text{open}} = -2\pi * \delta\Sigma^{\text{open}} = 2\pi * j \neq 0 , \tag{2.3}$$

where the relation $\delta\Sigma^{\text{open}} = -j$ is used. This relation just shows that Σ^{open} is nothing else but the world sheet of the electric Dirac string whose boundary is the electric current j (1-form). Clearly, if there is no external electric current, one must set $\Sigma^{\text{open}} = 0$. The inverse of the dual gauge coupling is denoted by $\beta_g = 1/g^2$, the strength of the self-interaction of the monopole field by λ, and the monopole condensate by v. These couplings are related to the mass of the dual gauge boson and the monopole mass as $m_B \equiv \sqrt{2/\beta_g}v = \sqrt{2}gv$ and $m_\chi = 2\sqrt{\lambda}v$, which determine not only the type of the superconductor vacuum through the so-called Ginzburg-Landau (GL) parameter $\kappa = m_\chi/m_B$, but also the thickness of the flux tube when the classical solution is considered. The value $\kappa < 1$ ($\kappa > 1$) describes the type-I (type-II) vacuum. Note that the DAH model is invariant under the transformation of fields $\chi \mapsto \chi \exp(i\theta)$, $B \mapsto B - d\theta$, when the U(1) dual gauge symmetry is not spontaneously broken.

The electric Dirac world sheet singularity which explicitly appears in the dual field strength (2.2) has the standard form and is required to satisfy the broken Bianchi identity (2.3). Clearly, such a singularity would give a divergent contribution in $(F)^2$ and must

dual gauge field	1-form	$B \equiv B_\mu dx_\mu$
monopole field	0-form	$\chi \equiv \phi \exp(i\eta)$
electric Dirac string	2-form	$\Sigma \equiv \frac{1}{2}\Sigma_{\mu\nu} dx_\mu \wedge dx_\nu$
electric current	1-form	$j \equiv j_\mu dx_\mu$

Table 2: Ingredients of the DAH model in the differential-form notation.

therefore be cancelled by a corresponding singular term in dB. Thus, it is useful to decompose the dual gauge field into two parts, the regular quantum part not containing an electric Dirac string and the singular part with an electric Dirac string, as

$$B = B^{\rm reg} + B^{\rm sing}\,, \tag{2.4}$$

where the singular part has the explicit form

$$B^{\rm sing} \equiv 2\pi\Delta^{-1}\delta * \Sigma^{\rm open}\,. \tag{2.5}$$

Here the inverse of the laplacian, Δ^{-1}, is the Coulomb propagator. Then, by using the relation $d\Delta^{-1}\delta + \delta\Delta^{-1}d = 1$ and the equation $\delta\Sigma^{\rm open} = -j$, we have

$$dB^{\rm sing} = 2\pi * \Sigma^{\rm open} + 2\pi\Delta^{-1}\delta * j = 2\pi(*\Sigma^{\rm open} + *C)\,, \tag{2.6}$$

where $*C$ is the 2-form field

$$*C = \Delta^{-1}\delta * j\,. \tag{2.7}$$

The dual field strength is, then, written as[1]

$$F = dB^{\rm reg} + 2\pi * C\,. \tag{2.8}$$

In the $q\bar{q}$ system, $*C$ turns out to contain the Coulomb electric field originating from the electric charges. In fact, we have

$$d * C = \Delta^{-1}d\delta * j = \Delta^{-1}\Delta * j = *j\,, \tag{2.9}$$

where we have used the electric current conservation condition, $\delta j = 0$. Note that the dual Bianchi identity for the dual gauge field $d^2B = 0$ is, of course, satisfied even after the decomposition into the regular and the singular parts, since we have $d^2B^{\rm reg} = 0$ and $d^2B^{\rm sing} = 2\pi(d * \Sigma^{\rm open} + d * C) = 2\pi(- * j + *j) = 0$. Thus we still have the relation $dF = 2\pi d * C = 2\pi * j$ as in eq. (2.3). Using the relation $\delta^2 = 0$ one can further show $\delta * C = 0$.

In the case that the phase of the monopole field is singular (multivalued), we also have closed electric Dirac strings $\Sigma^{\rm closed}$. This structure becomes manifest, if we write the phase with the regular and singular parts as

$$\eta = \eta^{\rm reg} + \eta^{\rm sing}\,, \tag{2.10}$$

[1]Note that the equality of F in eqs. (2.2), (2.8) does not at all mean that the Dirac string world sheet is simply replaced by the Coulomb electric field of the quark charges, since both objects are combined with different fields B and $B^{\rm reg}$ having different boundary conditions. After the decomposition (2.4) the Dirac string explicitly appears in the interaction term of eq. (2.12), where it dictates the boundary condition of the monopole field ϕ which has to vanish at the string core.

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where each part is defined so as to satisfy the relation

$$d^2\eta^{\rm reg} = 0\,, \qquad d^2\eta^{\rm sing} = 2\pi * \Sigma^{\rm closed} \neq 0\,. \tag{2.11}$$

Such a closed world sheet singularity can be regarded as the origin of a glueball excitation [30]. However, since this is not the issue of interest here, we neglect such singular phase contributions assuming that the phase is single-valued. Thus, from hereafter we simply use Σ as the world sheet of the open electric Dirac string, omitting the explicit label "open". The DAH action can then be written as

$$S(B,\chi,\Sigma) = \frac{\beta_g}{2}(dB^{\rm reg} + 2\pi * C)^2 + (d\phi)^2 + \left(\left(B^{\rm reg} + B^{\rm sing} + d\eta^{\rm reg}\right)\phi\right)^2 + \lambda\left(\phi^2 - v^2\right)^2\,. \tag{2.12}$$

3. Path integral transformation to Kalb-Ramond fields

In this section, in order to obtain the string representation, we shall next perform a field transformation in the path integral representation of the partition function of the DAH model given by

$$\mathcal{Z}(\Sigma) = \int \mathcal{D}B^{\rm reg}\delta[\delta B^{\rm reg} - f_B]\phi\mathcal{D}\phi\mathcal{D}\eta^{\rm reg}\exp\left[-S(B,\chi,\Sigma)\right], \tag{3.1}$$

where the DAH action has the form quoted in eq. (2.12). In the integral measure, we have inserted a usual gauge fixing term for the regular part of the dual gauge field $B^{\rm reg}$. The corresponding Faddeev-Popov (FP) determinant is omitted, since it contributes a trivial constant factor in an abelian theory. We start from the linearization of the square term $((B^{\rm reg} + B^{\rm sing} + d\eta^{\rm reg})\phi)^2$ by means of a 1-form auxiliary field E as

$$\begin{aligned}\exp&\left[-\left((B^{\rm reg} + B^{\rm sing} + d\eta^{\rm reg})\phi\right)^2\right]\\ &= \phi^{-4}\int \mathcal{D}E\exp\left[-\left\{\left(E,\frac{1}{4\phi^2}E\right) - i\left(E, B^{\rm reg} + B^{\rm sing} + d\eta^{\rm reg}\right)\right\}\right],\end{aligned} \tag{3.2}$$

so that the integration measure of the modulus of the monopole field ϕ in eq. (3.1) is modified as $\phi\mathcal{D}\phi \times \phi^{-4} \to \phi^{-3}\mathcal{D}\phi$. Then, based on the relation $(E, d\eta^{\rm reg}) = (\delta E, \eta^{\rm reg})$, we can integrate over the regular part of the phase $\eta^{\rm reg}$, which leads to the delta functional $\delta[\delta E]$ in the integration measure. The constraint on E can be resolved by introducing the 2-form Kalb-Ramond (KR) field h as

$$\delta[\delta E] = \int \mathcal{D}h\delta[\delta h - f_h]\delta[E - \delta * h]\,, \tag{3.3}$$

where the so-called hyper-gauge fixing delta functional appears in order to avoid the over-counting in the integration over h, which is due to the hyper-gauge invariance $h \mapsto h + d\Lambda$ with 1-form field Λ. The corresponding FP determinant is again omitted due to the same reason as for the dual gauge field. Now, we can immediately perform the integration over the auxiliary field E as

$$\mathcal{Z}(\Sigma) = \int \mathcal{D}B^{\rm reg}\delta[\delta B^{\rm reg} - f_B]\phi^{-3}\mathcal{D}\phi\mathcal{D}h\delta[\delta h - f_h] \times \\ \times \exp\left[-\left\{\frac{\beta_g}{2}(dB^{\rm reg} + 2\pi * C)^2 + (d\phi)^2 + \left(\delta * h, \frac{1}{4\phi^2}\delta * h\right) - \right.\right. \\ \left.\left. - i(\delta * h, B^{\rm reg} + B^{\rm sing}) + \lambda\left(\phi^2 - v^2\right)^2\right\}\right]. \tag{3.4}$$

Here, the kinetic term of the dual gauge field can be further rewritten as[2]

$$(dB^{\rm reg} + 2\pi * C)^2 = (dB^{\rm reg})^2 + 4\pi^2(j, \Delta^{-1}j)\,, \tag{3.5}$$

where the cross term $(dB^{\rm reg}, *C) = (B^{\rm reg}, \delta * C)$ vanishes due to the fact that $\delta * C = 0$. Moreover, other terms in the DAH action are also rewritten as

$$(*C)^2 = \left(\Delta^{-1}\delta * j, \Delta^{-1}\delta * j\right) = \left(j, \Delta^{-1}j\right), \tag{3.6}$$

$$\left(\delta * h, \frac{1}{4\phi^2}\delta * h\right) = \left(dh, \frac{1}{4\phi^2}dh\right), \tag{3.7}$$

$$(\delta * h, B^{\rm reg} + B^{\rm sing}) = (\delta * h, B^{\rm reg}) + 2\pi(h, \Sigma) + 2\pi(\delta h, \Delta^{-1}j)\,, \tag{3.8}$$

where the relation, $(*h, *C) = (\delta h, \Delta^{-1}j)$, has been taken into account. The partition function is then written as

$$\mathcal{Z}(\Sigma) = \int \mathcal{D}B^{\rm reg}\delta[\delta B^{\rm reg} - f_B]\phi^{-3}\mathcal{D}\phi\mathcal{D}h\delta[\delta h - f_h] \times \\ \times \exp\left[-\left\{\frac{\beta_g}{2}(dB^{\rm reg})^2 + 2\pi^2\beta_g(j, \Delta^{-1}j) + (d\phi)^2 + \left(dh, \frac{1}{4\phi^2}dh\right) - i(\delta * h, B^{\rm reg}) - \right.\right. \\ \left.\left. - 2\pi i(h, \Sigma) - 2\pi i(\delta h, \Delta^{-1}j) + \lambda(\phi^2 - v^2)^2\right\}\right]. \tag{3.9}$$

Next, the integration over the regular part of the dual gauge field $B^{\rm reg}$ is achieved in a standard way by an insertion of the identity

$$\text{const.} = \int \mathcal{D}f_B \exp\left[-\frac{\beta_g}{2\xi_B}f_B^2\right]. \tag{3.10}$$

Performing the integration over f_B and taking the Landau gauge $\xi_B = 1$, we get the terms

$$\frac{\beta_g}{2}(B^{\rm reg}, \Delta B^{\rm reg}) - i(\delta * h, B^{\rm reg}) = \frac{\beta_g}{2}\left(\left\{B^{\rm reg} - \frac{i}{\beta_g}\Delta^{-1}\delta * h\right\}, \Delta\left\{B^{\rm reg} - \frac{i}{\beta_g}\Delta^{-1}\delta * h\right\}\right) + \\ + \frac{1}{2\beta_g}\left(\delta * h, \Delta^{-1}\delta * h\right), \tag{3.11}$$

[2]Here and in other cases of partial integration, arising surface integrals are vanishing due to the vanishing of the regular fields at infinity.

where the last term can be rewritten as

$$(\delta * h, \Delta^{-1}\delta * h) = (h)^2 - (\delta h, \Delta^{-1}\delta h)\,. \tag{3.12}$$

Then, the gaussian integration over the shifted dual gauge field, $B^{\rm reg} - \frac{i}{\beta_g}\Delta^{-1}\delta * h \to B^{\rm reg}$, leads to the expression

$$\begin{aligned}\mathcal{Z}(\Sigma) = \int \phi^{-3}\mathcal{D}\phi\mathcal{D}h\delta[\delta h - f_h]\exp\Bigg[-\Bigg\{&2\pi^2\beta_g(j,\Delta^{-1}j) + (d\phi)^2 + \left(dh, \frac{1}{4\phi^2}dh\right) + \\ &+ \frac{1}{2\beta_g}(h)^2 - \frac{1}{2\beta_g}(\delta h, \Delta^{-1}\delta h) - 2\pi i(h,\Sigma) - \\ &- 2\pi i(\delta h, \Delta^{-1}j) + \lambda(\phi^2 - v^2)^2\Bigg\}\Bigg]\,. \end{aligned} \tag{3.13}$$

4. String representation

In this section, we aim to clarify the structure of the string representation of the DAH model. First, we divide the action into three parts as

$$S = S^{(1)} + S^{(2)} + S^{(3)}\,, \tag{4.1}$$

where each action is defined by

$$S^{(1)} = 2\pi^2\beta_g(j,\Delta^{-1}j)\,, \tag{4.2}$$

$$S^{(2)} = (d\phi)^2 + \left(dh, \frac{1}{4}\left\{\frac{1}{\phi^2} - \frac{1}{v^2}\right\}dh\right) + \lambda(\phi^2 - v^2)^2\,, \tag{4.3}$$

$$S^{(3)} = \frac{1}{4v^2}(dh)^2 + \frac{1}{2\beta_g}(h)^2 - \frac{1}{2\beta_g}(\delta h, \Delta^{-1}\delta h) - 2\pi i(h,\Sigma) - 2\pi i(\delta h, \Delta^{-1}j)\,, \tag{4.4}$$

respectively. The first term $S^{(1)}$ leads to the pure Coulomb potential (pure boundary contribution) when the static quark-antiquark system is investigated. The second term $S^{(2)}$ is defined so as to give a zero contribution to the effective string action in the case that $\phi = v$, which usually corresponds to taking the London limit $\lambda \to \infty$. In other words, this term leads for finite λ to a nonvanishing contribution due to the finite thickness (size of the core) of the string modelling the flux tube (ANO vortex), inside which the modulus of the monopole field smoothly becomes zero, $\phi = 0$. The third term $S^{(3)}$ is mainly responsible for the field contributions outside the core, near the surface of the flux tube, which remains even in the case that the monopole modulus has a constant value, $\phi = v$. It is interesting to evaluate the DAH action just on the string world sheet by taking into account the boundary conditions of the classical field equation. One finds that in order to get a finite energy contribution, we need to impose both $\phi = 0$ and $dh = 0$ on the string world sheet, where the second condition is resolved by $h = dA$ with 1-form field A. Then, by inserting this into eq. (4.4), we see that the contribution to the $S^{(3)}$ from the string world sheet is zero.

In order to discuss the effective string action definitely, let us consider the case that $m_B < m_\chi$. For the rough space-time structure, whose mesh size is larger than m_χ^{-1}, the London limit picture based on a dominating expression $S^{(3)}_{>m_\chi^{-1}}$ is valid, where the subscript means that one has to integrate over transverse distances from the string, $\rho > m_\chi^{-1}$, with the monopole mass m_χ chosen as an effective cut-off $\Lambda_{\rm eff}$. We will discuss the arbitrariness of choice of this effective cut-off later. Clearly, in this case, we cannot see the inside of the flux tube. On the other hand, to see the finer structure of the flux tube, variations of the monopole field in $S^{(2)}$ should be taken into account. The effective action of the vortex "core" contribution is described by $S^{(2)} + S^{(3)}_{<m_\chi^{-1}}$, and its leading term contains the Nambu-Goto action with the string tension $\sigma_{\rm core}$ and a current term $S_{\rm core}(j)$,

$$S_{\rm core} = S_{\rm core}(j) + \sigma_{\rm core} \int d^2\xi \sqrt{g(\xi)}\,, \tag{4.5}$$

where ξ^a $(a = 1, 2)$ parametrize the string world sheet described by the coordinate $\tilde{x_\mu}(\xi)$, and $g(\xi)$ is the determinant of the induced metric, $g_{ab}(\xi) \equiv \frac{\tilde{x}_\mu(\xi)}{\partial\xi^a}\frac{\tilde{x}_\mu(\xi)}{\partial\xi^b}$. The string tension $\sigma_{\rm core}$ is controlled by the solution of field equations derived from the action in the core region [31, 32]. Note that $S_{\rm core}(j)$ results only from $S^{(3)}_{<m_\chi^{-1}}$.[3]

Let us evaluate the string effective action of the "surface contribution" described by $S^{(3)}_{>m_\chi^{-1}}$. To do this, we first integrate out the KR field, and then extract the surface contribution from it by taking into account a suitable regularization in transverse variables. Since the action $S^{(3)}$ does not depend on the monopole modulus ϕ, the corresponding partition function is written as

$$\begin{aligned}\mathcal{Z}^{(3)} = &\int \mathcal{D}h\delta[\delta h - f_h] \times \\ &\times \exp\Bigg[-\bigg\{4\pi^2v^2\left(\Sigma, D\Sigma\right) + \frac{1}{4v^2}\left(\{h - 4\pi v^2 iD\Sigma\}, D^{-1}\{h - 4\pi v^2 iD\Sigma\}\right) - \\ &\qquad - 2\pi^2\beta_g\left(j, \left\{D - \Delta^{-1}\right\} j\right) - \\ &\qquad - \frac{1}{4v^2}\left(\{\delta h + 4\pi v^2 iDj\}, D^{-1}\Delta^{-1}\{\delta h + 4\pi v^2 iDj\}\right)\bigg\}\Bigg]\,.\end{aligned} \tag{4.6}$$

where we have defined the propagator of the massive KR field $D \equiv (\Delta + m_B^2)^{-1}$ and used the relation $(dh)^2 = (h, \Delta h) - (\delta h)^2$. The integration over the KR field is achieved in a similar way as for the dual gauge field by inserting an identity in form of a path-integral over the hyper-gauge fixing function f_h,

$$\text{const.} = \int \mathcal{D}f_h \exp\left[-\frac{1}{4v^2\xi_h}\left(\{f_h + 4\pi v^2 iDj\}, D^{-1}\Delta^{-1}\{f_h + 4\pi v^2 iDj\}\right)\right]. \tag{4.7}$$

Note that the integration over f_h and taking the hyper-Landau gauge $\xi_h = 1$, leads to a cancellation of the last term of the action in the partition function (4.6). Then, we

[3]If one approximately considers a vortex core with radius m_χ^{-1} in which $\phi = 0$ and $dh = 0$ everywhere as on the string world sheet, one finds $S_{\rm core}(j) = 0$, since $S^{(3)}_{<m_\chi^{-1}} = 0$.

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can integrate over the shifted KR field through the replacement $h - 4\pi v^2 iD\Sigma \to h$. The resulting effective string action from the surface contribution is then given by

$$S^{(3)}_{>m_\chi^{-1}} = 4\pi^2 v^2 \left(\Sigma, D\Sigma\right)\Big|_{>m_\chi^{-1}} + 2\pi^2\beta_g \left(j, \left\{D - \Delta^{-1}\right\} j\right)\Big|_{>m_\chi^{-1}}, \tag{4.8}$$

where "$|_{>m_\chi^{-1}}$" means that a corresponding effective cutoff (mesh size) should be taken into account. One finds that the first term represents the interaction between world sheet elements of the electric Dirac string via the propagator of the massive KR field. In tensor form, this expression can be written as

$$(\Sigma, D\Sigma)\Big|_{>m_\chi^{-1}} = \frac{1}{2}\int d^4x \int d^4y \Sigma_{\mu\nu}(x) D(x-y) \Sigma_{\mu\nu}(y)\Big|_{>m_\chi^{-1}} \tag{4.9}$$

where

$$\Sigma_{\mu\nu}(x) = \int_\Sigma d^2\xi \sqrt{g(\xi)} t_{\mu\nu}(\xi) \delta^{(4)}(x - \tilde{x}(\xi)), \tag{4.10}$$

and $t_{\mu\nu}(\xi) = \frac{\epsilon^{ab}}{\sqrt{g(\xi)}} \frac{\partial \tilde{x}_\mu(\xi)}{\partial \xi^a} \frac{\partial \tilde{x}_\nu(\xi)}{\partial \xi^b}$ is an antisymmetric tensor which determines the orientation of the string world sheet Σ. It is important to note that the regularization is achieved by introducing transverse coordinates Ξ^k $(k = 3, 4)$, which parametrize the direction d perpendicular to the string world sheet [13]. In other words, points in euclidean space close enough to Σ are parametrized as $x = x(\xi, \Xi)$ [13, 26].

Then the massive KR propagator in (4.9) can be written as

$$\begin{aligned} D(x-y) &= (\Delta + m^2)^{-1} \delta^{(4)}(x(\xi, \Xi) - y(\xi', \Xi')) \\ &= \left(- \sum_{k=3,4} \frac{\partial^2}{\partial \left(\Xi^k\right)^2} + \Delta_\xi + m^2 \right)^{-1} \frac{1}{\sqrt{g(\xi)}} \delta^{(2)}(\xi - \xi') \delta^{(2)}(\Xi - \Xi'), \end{aligned} \tag{4.11}$$

where the laplacian on the string world sheet is defined by

$$\Delta_\xi = -\frac{1}{\sqrt{g(\xi)}} \partial_a g^{ab}(\xi) \sqrt{g(\xi)} \partial_b . \tag{4.12}$$

In this scheme, the coherence length of the monopole field m_χ^{-1} plays the role of an effective cutoff of the Ξ integral. By the Taylor expansion of the propagator of the KR field, we obtain the explicit form of the effective string action as

$$\begin{aligned} &4\pi^2 v^2 \left(\Sigma, D\Sigma\right)\Big|_{>m_\chi^{-1}} \\ &\quad = \sigma_{\mathrm{surf}} \int d^2\xi \sqrt{g(\xi)} + \alpha_{\mathrm{surf}} \int d^2\xi \ \sqrt{g(\xi)} g^{ab}(\xi) \left(\partial_a t_{\mu\nu}(\xi)\right) \left(\partial_b t_{\mu\nu}(\xi)\right) + \mathcal{O}(\Delta_\xi^2) . \end{aligned} \tag{4.13}$$

Here, the first term represents the Nambu-Goto action with the string tension

$$\sigma_{\mathrm{surf}} = \pi v^2 \ln \frac{m_B^2 + m_\chi^2}{m_B^2} = \pi v^2 \ln\left(1 + \kappa^2\right), \tag{4.14}$$

and the second term is the so-called rigidity term with the negative coefficient

$$\alpha_{\rm surf} = \frac{\pi v^2}{2}\left(\frac{1}{m_\chi^2+m_B^2} - \frac{1}{m_B^2}\right) = -\frac{\pi\beta_g}{4}\frac{\kappa^2}{1+\kappa^2} \qquad (<0)\,, \tag{4.15}$$

where $\kappa = m_\chi/m_B$ is the GL parameter. Note that the rigidity term appears as a first order contribution in the derivative expansion with the laplacian Δ_ξ. The second term of (4.8), which is induced from the boundary of the string world sheet, is evaluated in a similar "effective regularization" scheme, discussed below. Finally, by combining it with the pure Coulomb term $S^{(1)}$, we get the effective action

$$\begin{aligned} S_{\rm eff}(\Sigma) &= S(j) + (\sigma_{\rm core}+\sigma_{\rm surf})\int d^2\xi\sqrt{g(\xi)} + \\ &+ \alpha_{\rm surf}\int d^2\xi\,\sqrt{g(\xi)}g^{ab}(\xi)\left(\partial_a t_{\mu\nu}(\xi)\right)\left(\partial_b t_{\mu\nu}(\xi)\right) + \mathcal{O}(\Delta_\xi^2)\,, \end{aligned} \tag{4.16}$$

where the boundary (electric current) contribution is given by

$$S(j) = S_{\rm core}(j) + \frac{1}{2\beta_e}(j,\Delta^{-1}j) + \frac{1}{2\beta_e}\left(j,\{D-\Delta^{-1}\}\,j\right)\Big|_{>m_\chi^{-1}} \tag{4.17}$$

Note that the Dirac quantization condition $4\pi^2\beta_e\beta_g = 1$ ($eg = 4\pi$) is taken into account, where $\beta_e = 4/e^2$ and $\beta_g = 1/g^2$. Equations (4.16) and (4.17) are the main result of this paper.

Here we would like to mention the role of the effective cut-off $\Lambda_{\rm eff} = m_\chi$ for the evaluation of $S^{(3)}$. In fact, we can choose any scale to divide into the low and the high energy parts as $S^{(3)}_{<\Lambda_{\rm eff}^{-1}}$ and $S^{(3)}_{>\Lambda_{\rm eff}^{-1}}$. If $\Lambda_{\rm eff} = cm_\chi$ (where $c \neq 1$) is taken, not only the $\sigma_{\rm surf}$ in eq. (4.14) but also $S_{\rm core} \equiv S^{(2)} + S^{(3)}_{<\Lambda_{\rm eff}^{-1}}$ are changed. However, the final expressions (4.16) and (4.17) are not affected by the choice of $\Lambda_{\rm eff}$, since the changes in $\sigma_{\rm surf}$ is absorbed by $\sigma_{\rm core}$, and the change in the third term of eq. (4.17) is absorbed by $S_{\rm core}(j)$. Due to the fact that $S^{(2)}$ contributes only in the region at $\rho < m_\chi^{-1}$, the choice $\Lambda_{\rm eff} = m_\chi$ ($c = 1$) for $S^{(3)}$ turns out to be the most "effective" one, which we take in this paper.

It is interesting to discuss the boundary contributions of the string world sheet. In order to get an explicit form, let us evaluate them with the static electric current

$$j_\mu(x) = \delta_{\mu 0}\{\delta^{(3)}(\boldsymbol{x}-\boldsymbol{a}) - \delta^{(3)}(\boldsymbol{x}-\boldsymbol{b})\}\,, \tag{4.18}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are the positions of the electric charges (The charge e is already factorized out). Denoting the distance between electric charges as $r = |\boldsymbol{a}-\boldsymbol{b}|$, the static potential is given by

$$\begin{aligned} V(r) &= V_{\rm core}(r) - \frac{1}{4\pi\beta_e r} + \\ &+ \frac{1}{2\beta_e}\int_{p_\rho<m_\chi}\frac{d^3p}{(2\pi)^3}\left(1-e^{-i\boldsymbol{p}\cdot\boldsymbol{r}}\right)\left(1-e^{i\boldsymbol{p}\cdot\boldsymbol{r}}\right)\left[\frac{1}{\boldsymbol{p}^2+m_B^2} - \frac{1}{\boldsymbol{p}^2}\right], \end{aligned} \tag{4.19}$$

where p_ρ is the momentum in the transverse direction, perpendicular to $\boldsymbol{r}$. In the rough approximation that $\phi = 0$ in the whole core region (it also means $dh = 0$ in the core), we have $V_{\rm core} = 0$, due to the fact that $S_{\rm core}(j) = 0$, and we get the final expression for the potential

$$V(r) = -\frac{e^{-m_B r}}{4\pi\beta_e r}\left[1 - e^{-(\sqrt{m_\chi^2+m_B^2}-m_B)r} + e^{-(m_\chi-m_B)r}\right], \qquad (4.20)$$

where constant terms have been dropped. Clearly, this form, which is valid for $r > 1/m_\chi$, is not the pure Yukawa potential nor a Coulomb potential. However, it is interesting to note that in the London limit $m_\chi \to \infty$ ($\lambda \to \infty$), the potential reproduces the usual Yukawa potential. The complete potential includes, of course, the confining potential $V_{\rm conf} = (\sigma_{\rm core} + \sigma_{\rm surf})r$ arising from the Nambu-Goto action in (4.16).

5. Summary and conclusions

The present paper is a first attempt to extend earlier path-integral investigations of the string representation of the DAH model [13, 14, 15, 18] as much as possible beyond the London limit. Particular attention was given to the treatment of boundary terms related to the nonconfining part of the $q\bar{q}$-potential. In fact, for a vortex with finite thickness and a vanishing monopole field inside the core, the usual expression for the Yukawa potential is expected to become modified. In order not to exclude from the very beginning even the possible appearance of Coulomb interactions (as indicated in phenomenological applications), we found it convenient to use a particular field decomposition, where the standard Dirac string describing the external quark-antiquark source is cancelled just keeping a Coulomb term which satisfies the broken dual Bianchi identity. For the proper treatment of boundary terms, we found it very convenient to use differential form techniques which, after performing a transformation to antisymmetric KR fields, required a careful treatment of corresponding gauge conditions for performing necessary path integrations. The investigation of the effective action requires the introduction of a cutoff. For this aim, analogously to refs. [13, 26], we reparametrized the integration variables x, y in eq. (4.9) near the string world sheet Σ in coordinates longitudinal and orthogonal to it, and used the monopole mass m_χ as an effective cutoff in the resulting transverse momentum integrals. The derivative expansion of the effective action then leads to the Nambu-Goto action and a rigidity term, with expressions for the (finite) string tension and the negative rigidity coefficient formally close to those of the London limit. Obviously, the string tension now gets an additional contribution from the vortex core which has to be calculated by using the classical field equations. Finally, concerning the nonconfining potential, there arises in the chosen regularization an interesting cancellation of the Coulomb term, originally appearing in the dual field strength in eq. (2.8), by a corresponding term of the boundary contribution of the KR field leaving instead a modified Yukawa interaction. In conclusion, we remark that the extension of these investigations to the more realistic SU(3_c)-DAH model is now under further investigation.

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